

→ for every f in D , if $U \circ f$ is bounded, then

$$\Phi(f) = \int_s (U \circ f)(s) dP(s)$$

So Suppes gets a measurable util. for actions, and a prob. measure over states of nature. But:

- 1) this does depend on defining $\text{prob} = \frac{1}{2}$ by the particular behavioristic method given.
- 2) It rules out (by A.9) such choice rules as minimax; as irrational. On what grounds?
3. My counterexample applies.
4. As Suppes admits, it leaves open the question as to how the man picks the a priori dist., or what is best.

S's points: Increasing interest in subjective prob. is based on awareness that the foundations of statistics should be constructed on the basis of a general theory of decision-making. In decision situations, subjective elements enter: i) utility for. on consequences.
? ii) a priori prob. dist. on states of nature; iii) determination of other prob. dists.

Both Savage + Suppes show that "if certain structure axioms are satisfied, any rational man acts as if he had an a priori 6.72 dist. on the states of nature." But certain of the "rational" axioms are also questionable. (aside from the normative problem of picking the best a priori dist.) Both want to base the theory of decision on behaviorally observable choices. Landable - but is it possible?

Suppes Winet: An Axiomatization of Utility Based on the Notion of Utility Differences

Management Science, Vol. 1, April-July 1955

Motivation: to separate utility and subjective probability.

The axioms derive measurable utility from the notion that differences can be compared (somehow) and equated.

Separate problem: to propose a physical operation leading to the comparison of differences

a) a 50-50 chance.

b) Man ~~with~~ owning (x, y) will give as much to have x replaced by z ($z \succ x$) as to have y replaced by w ($w \succ y$). This assumes that utilities are independent.

XO

I don't think (as Suppes does) that ²⁶¹"There can be little doubt that the experiment just described would provide a means of empirically testing the hypothesis" of independence.

c) No need to be ashamed of introspective evidence.

"It is also our opinion that many areas of economic and modern statistical theory do not warrant a behavioristic analysis of utility." In 14

Davidson & Suppes: A Finitistic Axiomatization of Subjective Probability and Utility.

Econometrica: Vol. 24, No. 3, July 1956

Other axiom theories require an infinite number of choices to observe. In this, set of outcomes and their prob. combos. is finite.
2) Choices are never required between a sure-thing and a gamble.

As usual: "When the subject is indifferent between the options, the assumption that he chooses to maximize expected utility" (p. 265) "suggests" that subjective probs. are equal. [But what justifies the assumption?]

Axiom A10 "is to guarantee that a person satisfying the axioms acts as if he had a subjective probability for any chance event E independent of what particular states are connected with the occurrence of E or its complement."

$$\phi(x) = 4$$

$$\phi(y) = 8$$

$$\phi(y') = 8$$

$$\phi(v) = 12$$

$$\phi(v') = 12$$

$$\phi(u) = -8$$

$$\phi(u') = -8$$

$$\frac{8-4}{12+8} =$$

[Check this: it must be what rules out minimax; capable of giving empirical test whether people assign subjective probs or minimax.]

A10: If $r(x, y; u, v) = r(x', y'; u', v')$ and $x, v M(E) y, u$
and $x \neq v'$ and $y' \neq u'$, then $x', v' M(E) y', u'$.

Suppes: problem of defining indifference when subject is always required to choose one alternative (anyway, we would expect random error near indifference).

Didn't allow subjects to give a judgment of indifference.

To get an interval scale, you have to have more than axioms of pure rationality, e.g. assumption of continuity and the existence of an infinite no. of probs (von Neumann; Suppes objects that this involves an infinite no. of observations).

-10 -5
-5 +5

$$pX + (1-p)Y$$

$$a+d = b+d$$

$$a+c = b+c$$

$$\begin{matrix} (a, d) & (a, c) \\ p & p \\ (b, d) & (b, c) \end{matrix}$$

$$(a, d) p (b, d)$$

$$(b, d) p (c, d)$$

$$(c, d) p (a, d)$$

[Theory as normative; why? What is it deduced from?]

$$(f, g) \equiv (f, g; \frac{1}{2}, \frac{1}{2})$$

Utility in Decision-Making

Suppes: Role of Subjective Probability, Edinb. Berkeley Symposium

General "subjectively estimates utilities" & probabilities.
 "He then (presumably) decides on f_1 or f_2 depending on which yields the greater expected utility..." Why? Why not consider variance, minimum, etc. (Because of the way utils were measured? This was not specified).

"The probabilities are measures of degree of belief, and the utilities measures of value. To be rational he should try to maximize expected value or utility w.r.t. his beliefs concerning the facts of the situation." Why?

✓ 4.3 $(f, g) \sim (g, f)$. "This guarantees that our special chance event has probability $\frac{1}{2}$." Like Savage, I take it that if we are indifferent between (f, g) and (g, f) , where if event E occurs we will take ^{the first} decision f and where E does not occur we will take the second, the 2 decisions not indifferent, then our subjective prob. for E is $\frac{1}{2}$.

I disagree. We may not have a subjective prob. for E , and are indiff. for this reason.

4.4 $f \succeq g \Leftrightarrow (f, h) \succeq (g, h)$ Sam's Indep. Axiom "An obvious

4.7 Arch ordering. "necessary to get measurability." substitution property.
 What if you don't have it?

I say this introduced by Friedman & Savage (ax. P3, p. 468)

4.8 "Constant decisions" — consequences independent of state of nature — exist corresponding to each consequence in C . I say this necessary, admits it doesn't have behavioural interpretation. Does it have any?

A.9. Sure-thing Principle. If $(f, g) \succ (f', g')$ ^{is better than} for every state of nature, then $(f, g) \succ (f', g')$.

"As Savage remarks, the kind of sure-thing principle expressed by this axiom is one of the most acceptable postulates of rational behavior."

A.11 Continuity.

"By and large, a structure axiom is an existential assertion."
 "A.1 - A.5 and A.9 are 'pure' rationality axioms which should be satisfied by any rational, reflective man in a decision-making situation."

		$\phi(f)$	$\phi(g)$	$\phi(h)$	$\phi(i)$								
		f	g	h	i	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*
$P(S_1)$	S_1	$\frac{U(x_1)}{x_1}$	$\frac{U(x_2)}{x_2}$	x_3	x_7	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$P(S_2)$	S_2	x_4	x_5	x_6	x_8	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8

$$i) (f, g) \succeq (h, i) \iff \phi(f) + \phi(g) \geq \phi(h) + \phi(i)$$

ii) ϕ unique up to linear trans. [This ϕ is a meas. util. for our actions]

Def: $U(x) = \phi(x^*)$. iii) $\exists$ a finitely additive prob. measure P on S

1. "C"-set of consequences — is an undefined primitive; so should include "payoff sets" — if Suppes will admit that such exist.
2. Why is axiom system necessary. Work with "Hypothesis" on measurable utility.

What Suppes experiments give is: predict a person's choices among lottery tickets offering a given, finite set of objects, as his subjective probability for ~~them~~ ^{the prizes} ~~changes~~ varies: on the basis of a finite no. of observations.

He does not predict: 1) choices among lotteries offering a different set of prizes (e.g. larger money amounts; political issues); 2) nature of preferences from non-risk data (supposing risk-data to be unavailable; e.g. from risk-data compiled from other persons similar in some respects — e.g. hot-rodgers, whose preferences & strength of preferences for cars is similar); 3) how subjective probabilities are derived from observable variables, e.g. objective frequencies, beliefs about laws of prob., etc. 4) hence, subjective probs for events other than the experimental events.

Limitation (1) can't be removed ⁱⁿ ~~by~~ his experimental setup. (2) can be investigated; but big differences between classes of people will probably only show up with larger (diff.) prizes. But 3) could be investigated very well, since size of prize doesn't matter as much; then 4) could be eliminated. unless probs turn out to depend on size or nature of prizes.

So maybe his experiment is best for investigating subjective prob.

